Avanti Learning Centres 2015 - 2017

P1. Units, Dimensions and Error Analysis

TABLE OF CONTENTS

P	1. Units, Dimensions and Error Analysis	1
	P1.1 Units and Dimensions	3
	P1.2 Significant Figures and Errors Analysis	8
	Additional Practice Problems	.15
	Answer Key	.19

PRE-REQUISITES

- 1. Conversion between basic units (millimeters to centimeters etc)
- 2. Ratio and proportion: The Unitary Method
- 3. Properties of Exponents and Scientific notation
- 4. Simultaneous equations

PRE-TEST (15 MINS)

- Q1. The value of $\frac{0.005}{1000}$ in scientific notation is _____.
- Q2. Write in decimal form (upto 2 decimal places)
 - A) $\frac{5}{4} =$
 - B) $\frac{4}{5} =$ ___
 - C) $\frac{10}{3} =$
- Q3. 20 % of 50 is _____
- Q4. Find the percent increase / decrease in each of the following cases:
 - A) The length of a rod changes from 4 cm to 6 cm
 - B) The weight of a person changes from 80 kg to 60 kg
- Q5. Compute the value of $\frac{6 \times 10^{-4}}{8 \times 10^4}$.
- Q6. If a humming bird flaps its wings once every 10^{-3} second, how many times does it flap its wings in a minute?
- Q7. Solve the equation x + y = 15; 4x y = 10
- Q8. Simplify the expression $\sqrt{A} \times A^6 =$

SCORE / 8

Score 1 point per correct answer. If you score less than 6, please revise the pre-requisite topics:

P1.1 Units and Dimensions

LEARNING OBJECTIVES

- 1. Identify different physical quantities and their representation using units and prefixes.
- 2. Define various standard system of units for measuring the 7 basic physical quantities
- 3. Classify units as fundamental and derived and to understand their relationship
- 4. Understand the concept of Dimensional Formula.
- 5. Illustrate the concept of dimensional analysis along with its applications

PRE-READING (30 MINUTES)

You may refer to one of the following sources:

Category	Book Name (Edition)	Chapter	Section
REQUIRED	NCERT Class XI Text book	1, 2	1.1-1.5,2.1-2.5, 2.8,2.9, 2.10
ADDITIONAL	H C Verma Volume I	1	1.3 and 1.4 (Important)

PRE-READING EXERCISE (20 MINUTES)

- Q1. What is the SI and CGS unit of Length, Mass, Time?
- Q2. What is the dimension of length, mass, time, speed?
- Q3. What is the SI unit used to denote luminous intensity?
- Q4. Angular Momentum is defined as *Mvr* where *M* is the mass, *v* is the velocity and *r* is the distance of the body from the axis. Write the dimensional formula for angular momentum.
- Q5. The mass of the earth is 6×10^{24} kg. The average mass of the atoms that make up Earth is 40 amu (i.e. atomic mass units). How many atoms are there in earth $(1 \ amu \approx 1.5 \times 10^{-27} Kg)$?

IN-CLASS EXERCISES (40 MINS)

SUBJECTIVE QUESTIONS

- Q1. Name three different systems of units which were used earlier? Also explain the SI system of units.
- Q2. State the Principle of Homogeneity of Dimensions.
- Q3. Do all physical quantities have dimensions? If no, name any four such quantities.
- Q4. Which of the following is a unit of distance (Explain):
 - i. Second
 - ii. Light year
 - iii. CSL (Chandrasekhar Limit)

LEVEL 1

- Q1. Find the dimensional formula of the following quantities
 - A) $Charge = Current \times Time$
 - C) Acceleration = Rate of change of velocity
 - E) Force = Mass \times Acceleration
 - G) Stress = $\frac{Force}{Area}$

Also find the SI units of these quantities.

- Q2. Convert the following units as mentioned
 - A) 2 decimeters = ____millimeters
 - C) $1 \mu g = _{kg}$
 - E) 100 degrees = _____radians
 - G) 2 nanoseconds = ____milliseconds

- B) Velocity = $\frac{\text{Straight line distance}}{\text{Time Taken}}$
- D) Momentum = $Mass \times Velocity$
- F) Strain = $\frac{\text{Change in length}}{\text{Original length}}$
- B) 1 Angstrom = ____kilometers
- D) 1 degree = ____minutes
- F) $1 \text{ day} = \underline{\hspace{1cm}} \text{seconds}$
- H) $15 \text{ fm} = ___ \text{nm}$
- Q3. (Compulsory PSV) Let us consider the equation $\frac{1}{2}mv^2 = mgh$, where m is the mass of the body, v is it's velocity, g is the acceleration due to gravity and h is the height. Is the equation dimensionally correct?

LEVEL 2

- Q4. [PSV] The SI unit of energy is J (Joule) which is also $kg m^2 s^{-2}$; that of speed v is ms^{-1} and of acceleration a is ms^{-2} . Which of the formulae for kinetic energy (K) given below can you rule out on the basis of dimensional arguments (m stands for the mass of the body):
 - A) $K = m^2 v^3$

B) $K = \frac{1}{2}mv^2$

C) K = ma

D) $K = \frac{3}{16}mv^2$

- E) $K = \frac{1}{2}mv^2 + ma$
- Q5. [PSV] The displacement of a progressive wave is represented by $y = A \sin(kx \omega t)$, where x is distance and t is time. Write the dimensional formula of (i) A and (ii) k (iii) ω . (Also write their SI units)
- Q6. (Compulsory PSV) The equation of state for a real gas can be expressed as

 $(P + \frac{a}{V^2})(V - b) = cT$ Where P is the pressure $(\frac{Force}{Area})$, V the volume, T the absolute temperature and a, b and c are constants. What are the dimensions of a, b and c?

Q7. The velocity of a wave in a string is dependent on the Tension in the string (T) and the mass per unit length of the string μ . Find the expression for velocity of the wave in the string as a function of some power of the Tension and the Mass per unit length of the string.

LEVEL 3

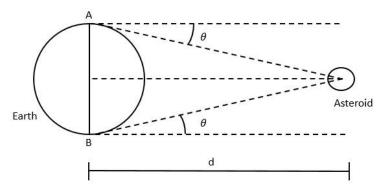
- Q8. [PSV] If momentum ($P = mass \times velocity$), area (A) and time (T) are taken to be fundamental quantities, then $kinetic\ energy = \frac{1}{2}mv^2$ has the dimensional formula
 - A) $(P^1A^{-1}T^1)$

B) $(P^2A^1T^1)$

C) $(P^1A^{-1/2}T^1)$

D) $(P^1A^{1/2}T^{-1})$

Q9. (Compulsory PSV) The angle subtended by an asteroid to the eye of two telescopes is measured to be 3" each from two observatories placed at diametrically opposite ends to the earth (As shown in the given Figure). If the radius of earth is 1.3×10^7 m, compute the distance of the asteroid from earth in AU (Calculate to 1 decimal place). (60" = 1' and 60' = 1 degree)($1 AU = 1.5 \times 10^{11} m$)



GROUP DISCUSSION (5 MINS)

Please go over each question in your groups and try to arrive at an answer together. After your discussion fill in the self-assessment below

SELF ASSESSMENT

Total Qs.		Correctly solved	Attempted	Not Attempted
6	Individually			
6	After GD			

Once your discussion is complete, please look at the solutions to this section at the back of the book and see if you can understand the rationale for each answer. Please make a note of any doubts at the end of this booklet. These doubts will be covered in your tutorial session.

HOMEWORK

SUBJECTIVE QUESTIONS

- Q1. Define Parsec unit. Compute 1 parsec into meters. (Given: 1A.U. = $1.496 \times 10^{11} m$).
- Q2. Name any two methods which are used to measure long distance.
- Q3. List some practical units of length for large distance.

LEVEL 1

- Q1. Fill in the blanks
 - A) $2 J/mol = \underline{\hspace{1cm}} \mu J/kmol$
 - B) $5 kg m s^{-2} = g cm s^{-2}$
 - C) 4J/kgK = MJ/g mK (MJ = mega-Joule, mK = milli-kelvin)
 - D) $12 Nm/s = \underline{\hspace{1cm}} \mu N cm/minute$

- E) $2 kg/mm^3 = ___g/m^3$
- F) $3m^2 = ___ \mu m^2$
- Q2. 15 kg/s = g/s = g/hour = kg/hour
- Q3. $4 L = ___ cm^3 = __ mm^3$
- Q4. If one light-year is approximately equal to 9.46×10^{15} m, then express 25 light years in centimeters (up to 3 decimal places).
- Q5. The mass of a solid cube of edge length 5 cm is 800 g. Determine the density (mass/volume) ρ of the cube in SI units.
- Q6. A vehicle is moving with a speed of 80 km/h. How many meters will it cover in 1 s? (Give answer to the nearest meter)
- Q7. The micrometer $(1 \mu m)$ is often called as the micron. (a) How many microns make up 500 km? (b) What is the fraction of a millimeter that equals $0.001 \mu m$?
- Q8. Suppose your hair grows at the rate of 0.17 inches per day. Find the rate at which it grows in nanometers per second. (Find answer to the nearest nanometer) $(1 inch = 2.54 cm) (2.54 \times 1.7 \approx 4.32)$
- Q9. The sun's angular diameter as measured from some far off planet is $8 \times 10^{-6} \ rad$. The diameter of the sun is $1.39 \times 10^{9} \ m$. What is the distance of the planet from the sun?
- Q10. Surface tension is defined as force exerted per unit length by water surface to the surrounding water and to the surface it contacts. This is responsible for circular shape of bubble and formation of drops on glass. Surface tension of water is 72 *dyne/cm*. Convert this quantity in SI units. (1 Dyne = $1 g cm/s^2$)
- Q11. The electric field due to a charge is defined as force (F) divided by the magnitude of charge (q). What is the dimension of electric field?
- Q12. What is the dimension of electric potential (V) due to a charge q, if $V = E \times r$. Where E is the electric field and r has a unit of distance. (For dimension of E, refer question above).
- Q13. The radius (r) of a circle inscribed in any triangle whose sides are a, b and c is given by the formulae $r = \left[\frac{(s-a)(s-b)(s-c)}{s}\right]^{\frac{1}{2}}$ where s is an abbreviation for semi-perimeter, $s = \frac{a+b+c}{2}$ Check this formula for dimensional consistency.
- Q14. $Electric\ Field\ = \frac{Force}{charge}$ and $Electric\ flux\ = Electric\ Field\ imes\ Area.$ Find the dimension of Electric flux.
- Q15. The energy density of the electrostatic field is given by $u = \frac{1}{2}\epsilon_0 E^2$, where E is the electric field and $\epsilon_0 = 8.85 \times 10^{-12} A^2 s^4 k g^{-1} m^{-3}$, find the dimension of u.

LEVEL 2

- Q16. The displacement of a particle is given by $s = at + \frac{b}{t^2}$ where a and b are constants, and t is the time. What are the dimensions of a and b?
- Q17. The speed (v) of ripples on the surface of water depends upon the surface tension $\left(\sigma = \frac{Force}{Length}\right)$, $density(\rho) = \frac{Mass}{Volume}$ and wavelength(λ). The speed v is proportional to which of the following; (the units of surface tension are N/m).
 - A) $\sqrt{\frac{\sigma}{\rho\lambda}}$

B) $\sqrt{\frac{\rho}{\sigma\lambda}}$

C) $\frac{\lambda}{\rho\sigma}$

D) ρλσ

Q18. Newton's law of universal gravitation is represented by $F = \frac{GMm}{r^2}$ Here F is the Gravitational force, M and m are masses, and r is a length. Force has the SI units $kg - m/s^2$. What is the SI unit of the proportionality constant G?

LEVEL 3

- Q19. The velocity of a particle is given by $v = abt^2 + \frac{a}{t}$ where a and b are constants and t is the time. What is the dimension of b?
- Q20. If the centripetal force is of the form $m^a v^b r^c$, find the values of a, b and c.
- Q21. The heat produced in a wire carrying an electric current depends on the current, the resistance and the time. Derive an equation relating the quantities using dimensional analysis. The dimensional formula of resistance is $ML^2I^{-2}T^{-3}$ and heat is a form of energy.
- Q22. Consider a simple pendulum, having a bob attached to a string that oscillates under the action of the force of gravity. Suppose that the time period of oscillation of the simple pendulum depends on its length (1), mass of the bob (m) and acceleration due to gravity (g). Derive the expression for its time period using method of dimensional analysis.
- Q23. When a solid sphere moves through a liquid, the liquid opposes the motion with a force F. The magnitude of F depends on the coefficient of viscosity η of the liquid, the radius r of the sphere and the speed v of the sphere. Assuming that F is proportional to different powers of these quantities, derive a formula for F using the method of dimensions. {Given: Dimension of coefficient of viscosity is $[M^1L^{-1}T^{-1}]$ }.
- Q24. Einstein discovered an important relation in his theory of relativity. However, when he wrote the relation, he forgot to put the constant c, the speed of light in the equation. What are the possible places the constant could be inserted in the equation?

$$l = l_o (1 - v^2)^{\frac{1}{2}}$$

In the above equation, l is the new length, l_o is the original length and v is the velocity of the body.

P1.2 Significant Figures and Errors Analysis

LEARNING OBJECTIVES

- 1. Identify the significant digits and rounding off and its application in arithmetic operations.
- 2. Identify different types of errors in measurement of physical quantities.
- 3. Understand the concept of errors and the terms associated with it. (accuracy and precision)
- 4. Compute the maximum error resulting from a combination of one or more physical quantities in mathematical operations.

PRE-READING (30 MINS)

You may refer to one of the following sources:

Category	Book Name (Edition)		Section
REQUIRED	NCERT Class 11 text book	2	2.6 , 2.7
REQUIRED	REQUIRED HC Verma Part 1 (Class 11) Textbook		2.12,-2.14
OPTIONAL	Maharashtra HSC Class XI Textbook	1	1.8

PRE READING EXERCISES (20 MINS)

- Q1. Round off 126.3 to the nearest integer
- Q2. Round off 12.92 to the nearest integer
- Q3. The scale used to measure the length of a football ground has a misprint on it. What type of error is this?
- Q4. My voltmeter shows an unpredictable fluctuation. What type of error are we encountering in this case?
- Q5. How many significant digits are there in 1.25

A) 1

B) 2

C) 3

- D) 4
- Q6. If you try, then it is possible that you can exactly measure the length of a table. (T/F)
- Q7. If the length of a table is 2 *m* and the measurements came to be 2.05, 2.03, 2.02, 1.98 *m* then what is the average error in the measurement.
- Q8. If Δa denotes the error in measuring a and Δb denote the error in measuring b then what is Δy if:

A) y = a + b

B) y = a-b

IN-CLASS EXERCISE (40 MINUTES)

SUBJECTIVE QUESTIONS

- Q1. Define the main types of errors.
- Q2. Explain how relative error is different from systematic error. Give atleast 3 points to explain your answer.
- Q3. What are the different ways of expressing error? Explain.

Q4. Write the formulae for error in different combination of quantities (addition, subtraction, multiplication, division and exponent).

LEVEL 1

- Q1. [PSV] The values of length of a rod in experiment were measured to be $2.48\,m$, $2.46\,m$, $2.49\,m$, $2.50\,m$ and $2.48\,m$. Find the average length, average absolute error, relative error and percentage error. Express the result with an error limit.
- Q2. Round off the following numbers to 4 significant figures
 - A) 55.324

B) 11.125

- C) 29.835
- Q3. If the error in measuring a quantity X is 2%, compute the percentage error in
 - A) X^2

B) *X*⁵

C) $\frac{1}{X}$

- D) \sqrt{X}
- Q4. (Compulsory PSV) 5.74 g of a substance occupies 1.2 cm^3 . Calculate it's $density = \frac{Mass}{Volume}$ to the correct number of significant digits (Use calculator to compute the fraction).

LEVEL 2

Q5. (Compulsory PSV) Listed below are a few physical quantities that depend on three fundamental quantities A, B and C in different ways. The percent errors in measuring A, B and C are respectively 1 %, 3 % and 4 % respectively. Find the percentage error in the resultant quantities

Quantity	Percent Error in the quantity
$W = \frac{A^{\frac{1}{3}}B}{C^3}$	
$X = \sqrt{ABC}$	

- Q6. [PSV] If displacement of a body $s=(200\pm5)$ m and time taken by it $t=(20\pm0.2)$ seconds, then find the percentage error in the calculation of average velocity.
- Q7. Indicate the type of systematic error likely to occur in each of the following situations:

	Situation	Type of Error
A)	The markings of the spring balance between 50 – 55 N have been erased due to corrosion	
B)	Out of habit, you always look at the nearest reading to the right instead of looking at the nearest reading to the left.	
C)	While measuring an angle using a protractor the angle's vertex is not made to coincide with the central point of the protractor	
D)	Both your eyes do not perceive the same image of the scale reading due to an eye defect.	

E)	An instrument for measuring the acceleration due to gravity consistently shows g = $10.1 \ m/s^2$	
F)	You do not stand at the center of a weighing machine when you try to record your weight	

LEVEL 3

Q8. (Compulsory PSV) Distance traveled by a particle is defined as $s = ut + \frac{1}{2}at^2$ where u is velocity a is acceleration, t is defined as time taken for motion. The value of the quantities comes to be $u = 5 \, m/s$, $t = 10 \, s$ and $a = 2 \, m/s^2$. In an experiment the error in measuring velocity, time and acceleration was 1 %, 1 % and 2 % respectively. Find the value of s and percentage error in calculation of distance (s) to correct significant digits.

GROUP DISCUSSION 2 (5 MINS)

Please go over each question in your groups and try to arrive at an answer together. After your discussion fill in the self-assessment below

SELF ASSESSMENT

Total Qs.		Correctly solved	Attempted	Not Attempted
r	Individually			
5	After GD			

Compulsory PSV are not included in self-assessment

Once your discussion is complete, please look at the solutions to this section at the back of the book and see if you can understand the rationale for each answer. Please make a note of any doubts at the end of this booklet. These doubts will be covered in your tutorial session.

HOMEWORK PROBLEMS

SUBJECTIVE QUESTIONS

- Q1. List and explain 2 different examples of systematic and random errors each.
- Q2. What is the difference between 5.0 and 5.00?
- Q3. Define Absolute, Relative and Percentage Error
- Q4. Derive the expression for relative error of AB if the absolute error and value of A and B is known.

LEVEL 1

- Q1. If the mass of a car is denoted as (200 ± 5) kg then:
 - A) What is the absolute error in measuring the car's mass?
 - B) What are the relative and percentage errors in measuring the car's mass?

- Q2. Find the number of significant digits in i) $0.0029 \, ii$) $1.9 \times 10^6 \, iii$) $12.900 \, iv$) $12900 \, iv$
- Q3. The masses of two objects are $20 \pm 2 kg$ and $10 \pm 4 kg$ respectively. Compute, with error, the value of
 - A) The total mass of the two objects
 - B) The difference between the masses of the two objects
- $Q4. \quad \mbox{ The error in measuring a quantity A is 5\%, quantity B is 2\%. Compute the maximum possible error}$
 - A) *AE*

B) A/B

Q5. Identify the more accurate instrument in each of the following cases (Assume that the error is only due to the instrument):

Actual measurement	Measurement by instrument 1	Measurement by instrument 2	Which is more accurate?
15 cm	14.36 cm	15.28 cm	
100 kg	102 kg	101 kg	
18.25 moles	18.65 moles	17.99 moles	

Q6. Identify the instrument with greater precision in each of the following cases:

Instrument 1	Precision	Instrument 2	Precision	Which is more precise?
A 50 cm scale with 100 small divisions		A 100 cm scale with 50 small divisions		
A weighing machine which can measure upto a milligram		A weighing machine which can measure upto a centigram		

Q7. For each of the following readings presented in the table given below, compute the absolute error in the reading, the relative error in the reading and the percent error.

True Value of measurement	Your observation	Absolute error	Relative error	Percent error
15 cm	14. 6 cm			
20 J	22 J			
3000 N	3050 N			

- A) Is the reading with the maximum absolute error also the one with the maximum relative error?
- B) Will this always be the case?
- Q8. Find the number of significant digits in
 - A) $0.2620 \ g/cm^3$

B) $2.031 N/m^2$

C) 0.0007083 m

D) $9.56 \times 10^{26} \, kg$

E) 55.40 J

- Q9. Round off each of the following numbers to 4 significant figures.
 - A) 14.645

B) 16.324

C) 10.335

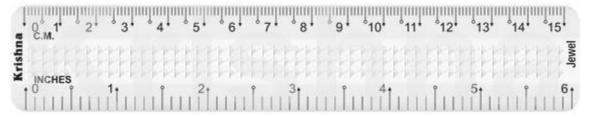
- Q10. Round off the following numbers to 3 significant figures
 - A) 7.230

B) 9.283

- C) 16.24
- Q11. Each side of a cube is 7.203 m in length. Compute the total surface area and the volume of the cube to appropriate significant figures. $(7.203^3 = 373.714754427)$
- Q12. Which of the following instruments to measure length is the most precise?
 - A) A Vernier caliper with 20 divisions on the sliding scale of length 2 cm
 - B) A screw gauge of pitch 1 mm and 100 divisions on the circular scale
 - C) An optical instrument that can measure length to within a wavelength of light (700 nm)
- Q13. Consider the following measurements of length by 5 different instruments and compute the absolute, relative and percent error for each.

Actual Measurement	Your observation	Absolute error	Relative error	Percent error
15 cm	14. 6 cm			
200 cm	215 cm			
1000 cm	990 cm			
5 m	4.8 m			
200 m	201 m			

- A) Which is the reading with the least absolute error?
- B) Which is the reading with the least relative error?
- Q14. Consider a standard measuring 15 cm ruler that all of you use.



- A) What is the least count if you use the centimeter side to measure length?
- B) What is the least count if you use the inch side? (1 inch = 2.54 cm)
- C) Which side is more precise for length measurements?

Q15. Shown below is some information on the measurement of a 10 cm long rod using 3 different instruments.

Instrument	Reading
1 meter scale with 1000 small divisions	10.1 cm
Vernier Caliper with least count 0.05 mm	10.15 cm
A laser scale capable of measuring lengths as low as 1 nm	10.2 cm

A) Which is the most precise instrument?

B) Which is the most accurate instrument?

- Q16. State the number of significant figures in
 - A) $77.01 N/m^2$

B) 0.0007095 m

C) 4.34000 J

D) 0.000708300 m

E) $0.390200 \ g/cm^3$

F) $0.04 m^3$

G) $0.019 m^3$

H) $0.9720 \ g/cm^3$

I) $3.034 N/m^2$

J) $6.79 \times 10^{29} \, kg$

K) 0.001320

- L) 1.2233500
- Q17. The sides of a rectangle are (10.5 ± 0.2) cm and (5.2 ± 0.1) cm. Calculate its perimeter with error limits.
- Q18. The measured values of the resistances of two resistors are (8 ± 0.3) ohm and (24 ± 0.5) ohm. The two resistors are connected in series such that $R_{total}=R_1+R_2$. Find the resistance of the combination. Also find the maximum percentage error.
- Q19. If the error in measurement of mass of a body be 3% and in the measurement of velocity be 2%. What will be maximum possible error in calculation of kinetic energy? (Kinetic energy is given as $K.E. = \frac{1}{2}mv^2$).

LEVEL 2

Q20. Two quantities *X* and *Y* are combined in various ways to result in new quantities.

 $X = (50 \pm 2)$ units

 $Y = (100 \pm 4) \text{ units}$

Quantity	Mean Value	Error	Value with Error	Relative error
X + 2Y				
X/Y				
XY				
3 <i>X</i> - 2 <i>Y</i>				
2XY				
4 <i>X</i> / <i>Y</i>				

- Q21. A physical quantity P is related to four observables a, b, c and d as $P = \frac{a^3b^2}{\sqrt{c}d}$. The percentage errors of measurement in a, b, c and d are 1%, 3%, 4% and 2%, respectively. What is the percentage error in the quantity P?
- Q22. The coefficient of viscosity (η) of a liquid by the method of flow through a capillary tube is given by the formula $\eta = \frac{\pi R^4 P}{8lQ}$ where R = radius of the capillary tube, l = length of the tube, P = pressure difference between its ends and Q = volume of liquid flowing per second. Which measurement needs to be made most accurately and why?
- Q23. The potential difference across a wire is measured with a voltmeter having least count 0.2 *volt* and the current in the wire is measured with an ammeter having least count 0.1 *ampere*. The following readings were obtained.

Voltmeter reading $(V) = 6.4 \ volt$

Ammeter reading (I) = 2.0 ampere

Find the value of the resistance of the wire with maximum error. Also find the maximum percentage error. Given that resistance $R = \frac{V}{I}$

- Q24. In an experiment for determining the density (ρ) of a rectangular block of a metal, the dimensions of the block are measured with calipers having a least count of 0.01 cm and its mass is measured with the beam balance of least count 0.1g. The measured values are: Mass of block (m) = 40.0g; Length of block $(x) = 4.0 \, cm$; Breadth of block $(y) = 2.50 \, cm$ and Thickness of the block $(z) = 0.40 \, cm$. Find the maximum permissible error in the determination of (ρ) . $\left(\rho = \frac{mass}{volume}\right)$
- Q25. Listed below are many physical quantities that depend on three fundamental quantities *A*, *B* and *C* in different ways. The percent errors in measuring *A*, *B* and *C* are respectively 1 %, 3 % and 4 % respectively.

Quantity	Percent Error in the quantity	
Q = AB/C		
R = ABC		
$U = A^2 B^3 C$		

- Q26. The percentage errors in the measurement of the length of a simple pendulum and its time period are 2% and 3% respectively. What is the maximum error in the value of the acceleration due to gravity? (Time period is given by $t = 2\pi \sqrt{\frac{l}{g}}$)
- Q27. A student measures the value of g with the help of a simple pendulum using the formula $g = \frac{4\pi^2 L}{T^2}$. The errors in the measurement of L and T are ΔL and ΔT respectively. Then in which of the following cases is the error in the value of g is minimum? (Given L = 100 cm T = 100 s)
 - A) $\Delta L = 0.5cm$, $\Delta T = 0.5s$

B) $\Delta L = 0.2cm$, $\Delta T = 0.2s$

C) $\Delta L = 0.1cm$, $\Delta T = 1.0s$

D) $\Delta L = 0.1 \, cm$, $\Delta T = 0.1 \, s$

Additional Practice Problems

Find the dimensional formula of each of the following quantities, given their definitions (Q 1 - Q 5):

- Q1. Angular Momentum (Angular momentum = mvr)
- Q2. Torque $(\tau = F \times r)$
- Q3. Pressure (*Pressure* = $\frac{F}{A}$)
- Q4. Kinetic energy (Defined as $\frac{1}{2}mv^2$)
- Q5. Potential energy (Defined as *mgh* where g is acceleration due to gravity)
- Q6. Verify whether each of the following is a correct statement
 - I. Time Period $T = \frac{mg}{L^2}$ (where m is mass, g is acceleration due to gravity, L is the length) is dimensionally correct.
 - II. Displacement $s = ut + \frac{1}{2}at^2$ (where u = initial speed or velocity, t = time, and a = acceleration) is dimensionally accurate
 - III. $v^2 = u^2 + 2as$ (where u = initial speed or velocity, v = final speed or velocity, a = acceleration, s = displacement) is dimensionally incorrect
 - IV. v = u + at (where v = final speed or velocity, u = initial speed or velocity, a = acceleration, t = time) is dimensionally correct
 - A) FTFT

B) FTTT

C) TTTT

- D) TTFT
- Q7. In the expression $X = 3YZ^2$, X and Z have dimensions of capacitance $[M^{-1}L^{-2}T^2Q^2]$ and magnetic induction $[MT^{-1}Q^{-1}]$ respectively. What is the dimension of Y in MKSQ system?
 - A) $[M^{-3}L^{-1}T^3O^4]$

B) $[M^{-3}L^{-2}T^4Q^4]$

C) $[M^{-2}L^{-2}T^4O^4]$

- D) $[M^{-3}L^{-3}T^4O]$
- Q8. A cube has a side of length $1.2 \times 10^{-2} m$. Calculate its volume
 - A) $2 \times 10^{-6} m^3$

B) $1.73 \times 10^{-6} m^3$

C) $1.7 \times 10^{-6} m^3$

- D) $1.728 \times 10^{-6} m^3$
- Q9. A cylindrical wire has mass (0.300 ± 0.003) g, radius (0.500 ± 0.005) mm and length (6 ± 0.06) cm. The maximum percentage error in the measurement of its density is
 - A) 1%

B) 2%

C) 3%

- D) 4%
- *Q10. A student uses a simple pendulum of exactly 1m length to determine *g*, the acceleration due to gravity. He uses a stop watch with the least count of 1 s for this and records 40 s for 20 oscillations, for this observation which of the following statement(s) is/are true
 - A) Error ΔT is measuring T, the time period, is 0.05s
 - B) Error ΔT is measuring T, the time period, is 1s
 - C) Percentage error in the determination of *g* is 5%
 - D) Percentage error in the determination of g is 2.5%
- Q11. A gas bubble, from an explosion under water oscillates with a period T proportion to $p^a d^b E^c$, where p is the static pressure, d is the density of water and E is the total energy of the explosion. What would the expression for time period be proportional to?
 - A) $\sqrt[2]{d} \sqrt[3]{E} p^{\frac{5}{6}}$

B) $\sqrt[2]{d} \sqrt[3]{E} p^{-\frac{5}{6}}$

C) $\sqrt[2]{E} \sqrt[3]{d} p^{-\frac{5}{6}}$

D) $\sqrt[2]{E} \sqrt[3]{d} p^{-\frac{5}{6}}$

Q12. Find the dimensional formulae of the following quantities:

- A) The universal constant of gravitation G,
- B) The surface tension S.
- C) The thermal conductivity k
- D) The coefficient of viscosity η

- P) *MT*⁻²
- 0) $M^{-1}L^3T^{-2}$
- R) $ML^{-1}T^{1}$
- S) $MLT^{-3}K^{-1}$
- T) $MLT^{-5}K^{-2}$

Some equations involving these quantities are

$$F=\frac{Gm_1m_2}{r^2}$$
 , $S=\frac{F}{I}$

$$Q = k \frac{A(T_2 - T_1)t}{d}$$
 and $F = -\eta A \frac{v_2 - v_1}{d_2 - d_1}$

Where the symbols have their usual meanings.

Q13. Find the SI-unit of

- A) The charge Q,
- B) The potential V,
- C) The capacitance C, and
- D) The resistance (R)

- P) $ML^2I^{-1}T^{-3}$
- Q) IT
- R) $ML^2I^{-1}T^{-3}$
- S) $M^{-1}L^{-2}I^2T$

Given that Potential = $\frac{Force \times distance}{Charge}$

Charge = $Current \times Time$

Charge = Capacitance \times Potential

Potential = $Current \times Resistance$

Q14. The SI and CGS units of energy are joule and erg respectively. How many ergs are equal to one joule?

A) 10^3

B) 10^{-3}

C) 10^7

D) 10^{-7}

Q15. Young's modulus of steel is $19 \times 10^{10} N/m^2$. Express it in $dyne/cm^2$. Here dyne is the CGS units of force

A) 19×10^{10}

B) 19×10^{17}

C) 19×10^{11}

D) 19×10^{3}

Q16. If velocity, time and force were chosen as basic quantities then find the dimension of mass

A) FTV^{-2}

B) FTV

C) FT^2V^{-2}

D) FTV^{-1}

Q17. The distance covered by a particle in time t is given by $x = a + bt + ct^2 + dt^3$; find the dimensions of a, b, c and d.

- A) a
- B) *b*

- P) I
- Q) LT^{-3}

C) *c*

R) LT^{-2}

D) d

S) LT^{-1}

Q18. When a solid sphere moves through a liquid, the liquid opposes the motion with a force F. the magnitude of F depends on the coefficient of velocity η of the liquid, the radius r of the sphere and speed v of the sphere.

Assuming that F is proportional to different powers of these quantities, guess a formula for F using the method of dimensions

A)
$$F = k\eta r^2 v$$

C)
$$F = kn^2rv$$

B)
$$F = k\eta r v^2$$

D)
$$F = k\eta rv$$

Q19. The time period of a block attached to a spring, undergoing simple harmonic motion is dependent on the mass of the block (m) and the spring constant (k). Assuming that the time period of the block is proportional to some power of mass (m) and the spring constant (k), find the expression for the time period of the block using dimensional analysis. It is given that the dimension of the spring constant is [M¹L⁰T-²].

A)
$$T = \sqrt{\frac{m}{k}}$$

B)
$$T = \frac{\sqrt{m}}{k}$$

C)
$$T = constant \sqrt{\frac{m}{k}}$$

D)
$$T = constant \sqrt{\frac{k}{m}}$$

Q20. If velocity (V), force (F) and energy (E) are taken as fundamental units, then dimensional formula for mass will be

A)
$$V^{-2}F^0E^1$$

B)
$$V^0F^1E^2$$

C)
$$V^1F^{-2}E^0$$

D)
$$V^{-2}F^0E^1$$

Q21. Time period T of a simple pendulum may depend on m, the mass of the bob, l, the length of the string and g, the acceleration due to gravity, i.e. $T = m^a l^b g^c$. What are the values of a, b and c?

A)
$$a = 0, b = \frac{1}{2}$$
 and $c = -\frac{1}{2}$

B)
$$a = 0, b = \frac{1}{2}$$
 and $c = \frac{1}{2}$

C)
$$a = \frac{1}{2}$$
, $b = \frac{1}{2}$ and $c = -\frac{1}{2}$

D)
$$a = 0, b = -\frac{1}{2}$$
 and $c = -\frac{1}{2}$

022. For each of the following, cases provided in the table below.

A) The reading with the maximum absolute error is also the one with the maximum relative error. (T/F)

B) This will always be the case. (T/F)

True Value of measurement	Your observation
30 m/s ²	29. 4 m/s ²
800 amp	900 amp
500 μC	540 μC

Q23. If the time period of oscillation t of a drop of liquid of density d, radius r, vibrating under surface tension s is given by the formula $t = \sqrt{d^a r^b s^c}$ and a = 1, c = -1 then what is the value of b?

A) 1

B) 3/2

C) 3

D) 2

Q24. An astronomical unit (AU) is the average distance between the earth and the sun, approximately measured to be 1.5×10^8 km. If the speed of light is a constant 3.0×10^8 m/s calculate the speed of light in terms of astronomical units per minute.

A) 0.002

B) 0.12

C) 0.2

D) 1.2

- Q25. If the error in the measurement of the volume of a sphere is 6% then the error in the measurement of its surface area is
 - A) 2 %

B) 3 %

C) 4 %

- D) 6%
- Q26. If the centripetal force is of the form $m^a v^b r^c$ then find the values of a, b, and c
 - A) a = 1, b = 0, c = -2

B) a = 1, b = 2, c = -1

C) a = 1, b = 0, c = -1

D) a = 1, b = -2, c = -2

Answer Key

PRE-TEST

- Q1. 5×10^{-6}
- Q2. A) $\frac{5}{4} = 1.25$
 - B) $\frac{4}{5} = 0.8$
 - C) $\frac{10}{3}$ = 3.33
- Q3. 10

- Q4. A) 50 % increase B) 25 % decrease
- Q5. 7.5×10^{-9}
- Q6. 6×10^4 times
- Q7. x = 5, y = 10
- 08. $A^{\frac{13}{2}}$

P1.1 UNITS AND DIMENSIONS

PRE-READING EXERCISE

- Q1. SI: meter, kilogram, second, CGS: centimeter, gram, second
- Q2. $Length = [L], Mass = [M], Speed = [M^0L^1T^{-1}]$
- Q3. Candela
- Q4. $M^1L^2T^{-1}$
- Q5. 10^{50} atoms

IN-CLASS EXERCISE

- Q1. A) Charge = $M^0L^0T^1A^1$, A s (Also called Coulomb)
 - B) $Velocity = M^0L^1T^{-1}, m/s$
 - C) Acceleration = $M^0L^1T^{-2}$, m/s^2
 - D) Momentum = $M^1L^1T^{-1}$, Kg m/s
 - E) Force = ML^1T^{-2} , $Kg m/s^2$ (Also called Newton)
 - F) $Strain = M^0L^0T^0$, Unitless
 - G) $Stress = M^1L^{-1}T^{-2}$, $Kg/m/s^2$ (Also called Pascal)
- Q2.
- Q3. Yes
- 04. A, C, E
- Q5. i) ω is $[M^0L^0T^{-1}]$ SI Unit m^{-1}
 - ii) k is $[M^0L^{-1}T^0]$ SI Unit s^{-1}
- Q6. i) $a = [M^1 L^5 T^{-2}]$
 - ii) $b = [M^0 L^3 T^0]$
 - iii) $c = [M^1L^2T^{-2}K^{-1}]$
- Q7. I
- Q8. $v = k \sqrt{\frac{T}{\mu}}$
- Q9. 2.2 AU

HOMEOWRK

LEVEL 1

- Q1. A) $2 J/mol = 2 \times 10^9 \, \mu J/kmol$
 - B) $5 kg m s^{-2} = 5 \times 10^5 g cm s^{-2}$

- C) $4\frac{J}{kg\,K} = 4 \times 10^{-12} \frac{MJ}{g-mK}$ (MJ = mega-Joule, mK = milli-kelvin) $(4J = 4 \times 10^{-6} \, MJ \, and \, 1 \, kg^{-1} \, K^{-1}$ $= 10^{-6} \, g^{-1} mK^{-1}$)
- D) $12 \frac{Nm}{s} = 7.20 \times 10^{10} \frac{\mu N cm}{minute}$ $(1 N = 10^6 N, 1mz = 10^2 \text{ cm} \text{ and } 1 \text{ per second} = 60 \text{ per minute})$
- E) $2 kg/mm^3 = 2 \times 10^{12} g/m^3$
- F) $3 m^2 = 3 \times 10^{12} \mu m^2$
- Q2. $15000 \, g/s$, $5.4 \times 10^7 \, g$ /hour, $54000 \, kg$ /hour
- Q3. $4000 cm^3$, $4 \times 10^6 mm^3$
- Q4. $2.365 \times 10^{19} \ cm$
- Q5. $6400 \, kg/m^3$
- Q6. 22 m is covered
- Q7. (a) $5 \times 10^{11} \mu m$ (b) $\frac{1}{10^6}$
- Q8. $50\frac{nm}{s}$
- Q9. $1.7375 \times 10^{15} \ m$
- Q10. $72 \times \frac{10^{-3} kg}{c^2}$
- Q11. $[M^1L^1I^{-1}T^{-3}]$
- Q12. $[M^1L^2I^{-1}T^{-3}]$
- Q13. It is dimensionally Consistent
- Q14. $[M^1L^3I^{-1}T^{-3}]$
- Q15. $[M^1L^{-1}I^0T^{-2}]$

LEVEL 2

- Q16. $[a] = [M^0L^1T^2], [b] = [M^0L^1T^2]$
- Q17. A
- Q18. $\frac{m^3}{kg*sec^2}$
- Q19. $[M^0L^0T^{-3}]$
- Q20. a = 1, b = 2 and c = -1
- Q21. $h = constant \times I^2Rt$
- Q22. $T = C \times \sqrt{\frac{l}{g}}$
- Q23. $F = Constant \times \eta rv$
- Q24. c^2 as divisor of v^2

P1.2 SIGNIFICANT FIGURES AND ERRORS ANALYSIS

PRE READING EXERCISES

- Q1. 126
- Q2. 13
- Q3. Instrumental Error
- Q4. Random Error
- Q5. 3
- Q6. F
- Q7. 0.02
- Q8. In both cases $\Delta y = \Delta a + \Delta b$

IN-CLASS EXERCISE

- Q1. $L_{av} = 2.48 \ m$, $\Delta L_{ab} = 0.01 \ m$, $\Delta L_{rel} = 0.0042$, $\Delta L_{percentage} = 0.42 \ \%$, $L_{rod} = 2.48 \ \pm 0.01 \ m$
- Q2. A) 55.32
 - B) 11.13
 - C) 29.84

- Q3. A) 4%
 - B) 10%
 - C) 2%
 - D) 1%
- Q4. $4.8 \ g/cm^3$
- Q5. (1) Percentage Error in W = 15.33 %
 - (2) Percentage Error in X = 4%
- Q6. 3.5 %
- Q7. A) Instrumental
 - B) Personal
 - C) Experimental
 - D) Personal
 - E) Instrumental
 - F) Personal
- Q8. 3 %

HOMEWORK

- Q1. A) Absolute error 5 kg (Not 10 kg)
 - B) Relative and percentage errors? (0.025 and 2.5% respectively)
- Q2. i) 2 ii) 2 iii) 5 iv) 3
- Q3. A) $30 \pm 6 \, kg$
 - B) $10 \pm 6 \, kg$
- Q4. A) AB (7%)
 - B) A/B (7%)
- Q5. 1) Instrument 2
 - 2) Instrument 2
 - 3) Instrument 2
- Q6. 1) Instrument 1
 - 2) Instrument 1,
- Q7. A) No
 - No B) No
 - 1) 2.7%
 - 2) 10%
 - 3) 1.7%
- Q8. A) There are four significant digits. The zeroes on the right are significant but to the left of

- the first non-zero digit are not significant. Hence all the numbers after decimal are significant.
- B) All the four digits are significant.
- C) Last four digits (7, 0, 8 and 3) are significant but zeroes to the left of the first non-zero digit are not significant
- D) All the three digits are significant.
- E) All four digits are significant.
- Q9. A) 14.64
 - B) 16.32
 - C) 10.34
- Q10. A) 7.23
 - B) 9.28
 - C) 16.2
- Q11. $373.7 m^3$
- Q12. (C) optical instrument

Q13.

Actual Measurement	Your observation	Absolute error	Relative error	Percent error
15 cm	14. 6 cm	0.4 cm	0.02	2 %
200 cm	215 cm	15 cm	0.075	7.5 %
1000 cm	990 cm	10 cm	0.01	1.0 %

5 m	4.8 m	0.2 m	0.04	4 %
200 m	201 m	1 m	0.005	0.5 %

- A) The first reading with an error just 0.4 cm.
- B) The last reading with a relative error of only 0.005 (Although it has the highest absolute error)
- Q14. A) 1 mm
 - B) 0.1 inch (2.54 mm)
 - C) 6 Centimeter
- Q15. A) Laser Scale
 - B) Meter Scale
- Q16. A) All four digits are significant.
 - B) Last four digits are significant.
 - C) All the digits are significant. Hence there are 6 significant digits here.
 - D) Last six digits are significant
 - E) All the digits after the decimal are significant. Here there are 6
 - F) Only the last digit (4) is significant.
 - G) Last two digits 1 and 9 are significant.
 - H) All the four digits after the decimal are significant i.e. 9, 7, 2 and 0.
 - I) All the four digits are significant.
 - J) All the three digits are significant.
 - K) There are four significant digits.
 - L) All the eight digits are significant.
- Q17. 31.4 ± 0.6 cm
- Q18. 2.6 %
- Q19. 7%
- Q20. Relative errors (1) 0.04 (2) 0.08 (3) 0.08 (4) 0.28 (5) 0.08 (6) 0.08

Quantity	Mean Value	Error	Value with Error	Relative error
X + 2Y	250	10	250 ± 10	0.04
X/Y	0.5	0.04	0.5 ± 0.04	0.08
XY	5000	400	5000 ± 400	0.08
3 <i>X</i> – 2 <i>Y</i>	-50	14	-50 ± 14	0.28
2XY	10000	800	10000 ± 800	0.08
4X/Y	2	0.16	2 ± 0.16	0.08

- Q21. 13 %
- Q22. Radius (R) needs to be measured most accurately because it has the highest exponent
- Q23. 8%
- Q24. $0.3 \frac{kg}{m^3}$

- Q25. 1) 8%
 - 2) 8%
 - 3) 15 %
- Q26. 8%
- Q27. D

ADDITIONAL PRACTICE PROBLEMS - ANSWERS

- Q1. Angular Momentum = $ML^2T^{-1}[ML^2T^{-1}]$
- Q2. Torque = $[ML^2T^{-2}][ML^2T^{-1}]$
- Q3. Pressure= $[ML^{-1}T^{-2}]1$
- Q4. Kinetic Energy= $[ML^2T^{-2}]$
- Q5. Potential Energy= $[ML^2T^{-2}]$
- Q6. A
- Q7. B
- Q8. C
- Q9. D
- Q10. A, C
- Q11. B
- Q12. A-Q, B-P, C-K, D-R
- Q13. A-Q, B-P, C-S, D-R

- Q14. C
- Q15. C
- Q16. D
- Q17. A-P, B-S, C-R,D-Q
- Q18. D
- Q19. C
- Q20. A
- Q21. A
- Q22. A) True B) False
- Q23. a = 1, and c = -1, b = 3
- Q24. 0.12 AU/minute
- Q25. C
- Q26. B